TRIBHUVAN UNIVERSITY

Institute of Science and Technology

2065

Bachelor Level/ First Year/ First Semester/ Science — Full Marks: 80

Computer Science and Information Technology (MTH 104) — Pass Marks: 32

(Calculus and Analytical Geometry) — Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Attempt all the questions:

<u>Group A</u> (10x2=20)

1. Verify Rolle's theorem for the function $f(x) = \frac{x^3}{3} - 3x$ on the interval [-3, 3].
2. Obtain the area between two curves $y = \sec^2 x$ and $y = \sin x$ from $x = 0$ to $x = {}^{\pi}/_{4}$.
3. Test the convergence of p – series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for p > 1.
4. Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.
5. Find a vector perpendicular to the plane of P(1, -1, 0), C(2, 1, -1) and R(-1, 1, 2).
6. Find the area enclosed by the curve $r^2 = 4\cos 2\theta$.
7. Obtain the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (4, -5) if $f(x, y) = x^2 + 3xy + y - 1$.
8. Using partial derivatives, find $\frac{dy}{dx}$ if $x^2 + \cos y - y^2 = 0$.
9. Find the partial differential equation of the function $(x - a)^2 + (y - b)^2 + z^2 = c^2$.
10. Solve the partial differential equation $x^2p + q = z^2$.

<u>Group B</u> (5x4=20)

11. State and prove the mean value theorem for a differential function.

12. Find the length of the Astroid $x = \cos^3 t,\ y = \sin^3 t$ for $0 \leq t \geq 2\pi$.

13. Define a curvature of a curve. Prove that the curvature of a circle of radius a is 1/a.

14. What is meant by direction derivative in the plain? Obtain the derivative of the function $f(x, y) = x^2 + xy$ at P(1, 2) in the direction of the unit vector $v = \left(\frac{1}{\sqrt{2}}\right) i + \left(\frac{1}{\sqrt{2}}\right) j$.

15. Find the center of mass of a solid of constant density δ, bounded below by the disk: $x^2 + y^2 = 4$ in the plane $z = 0$ and above by the paraboloid $z = 4 - x^2 - y^2$.

Group C (5x8=40)

16. Graph the function $f(x) = -x^3 + 12x + 5$ for $-3 \leq x \leq 3$.

17. Define Taylor's polynomial of order n. Obtain Taylor's polynomial and Taylor's series generated by the function $f(x) = e^x$ at $x = 0$.

18. Obtain the centroid and the region in the first quadrant that is bounded above by the line $y = x$ and below by the parabola $y = x^2$.

19. Find the maximum and the minimum values of $f(x, y) = 2xy - 2y^2 - 5x^2 + 4x - 4$. Also find the saddle point if it exists.

OR

Evaluate the integral $\int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2-3y^2}^{6-x^2-y^2} dz\ dx\ dy$.

20. What do you mean by d' Alembert's solution of the one-dimensional wave equation? Derive it.

OR

Find the particular integral of the equation $(D^2 - D^1)z = 2y - x^2$ where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$.